ME 1401 FINITE ELEMENT ANALYSIS

UNIT I
PART -A

1. What is meant by Finite Element Analysis?

2. Why polynomial type of interpolation functions is mostly used in FEM?

3. What are 'h' and 'p' versions of Finite Element method?

4. Name the weighted residuals techniques.

5. What is meant by post processing?

6. Distinguish between essential boundary conditions and natural boundary conditions
7. Name any 4 FEA software's?

8. What are the general constituents of finite element software?

9. What is Rayleigh-Ritz method?

10. During discretization, mention the places where it is necessary to place a node?

PART B

1. Solve the simultaneous systems of equation using Gauss-Elimination method.

$3x + y - z = 3$; $2x - 8y + z = 3$; $x - 2y + 9z = 8$ (16)

2. The following differential equation is available for a physical phenomenon:

d2y – 10x2 = 5; $0 \leq x \leq 1$ dx2

The boundary conditions are: $y(0) = 0$ and $y(1) = 0$.By using Galerkin's method of weighted residuals, find an approximate solution of the above differential equation.(16)

3. A simply supported beam subjected to uniformly distributed load 'q' over entire span and also point load of magnitude 'P' at the centre of the span. Calculate the bending

moment and deflection at mid span by using Rayleigh-Ritz method and compare with the exact solution. (16)

4. A bar of uniform cross section is clamped at one end and left free at the other end and it is subjected to a uniformly distributed load of “q” over its entire run. Calculate the displacement and stress in the bar using three term polynomial. Compare with exact solutions. Also, determine the stresses and displacements if the bar is clamped at both ends. (16)

5. Determine a 2 parameter solution of the following using the Galerkin’s method and compare it with the exact solution. (16)

d2y/dx2 = - cos π x, $0 \le x \le 1$

u (0) = 0, u(1) = 0

6. Solve the following equation using a 2 parameter trial solution by:

a) Point collocation method.

b) Galerkin’s method.

dy/dx + y = 0, $0 \le x \le 1$, y(0) = 1 (16)

7. The following differential equation is available for a physical phenomenon.

d2y/dx2 +50 = 0, $0 \le x \le 10$

Trial function is y = a1x (10-x), y (0) = 0, y(10) = 0. Find the value of a1 using all the weighted residual techniques and compare the solutions. (16)

8. Solve the given equations by Gauss-Elimination method.

2x + 4y + 2z = 15; 2x + y + 2z = -5; 4x + y – 2z = 0 (6)

9. List the advantages, disadvantages and applications of FEM. (8)

10. Explain the various weighted residual techniques. (16)

UNIT II

PART A

1. Define shape function.

2. How do you calculate the size of the global stiffness matrix?

3. What are the characteristics of shape function?

4. Write down the expression of stiffness matrix for one dimensional bar element.

5. State the properties of stiffness matrix.

6. What is truss?

7. Define total potential energy.

8. State the principle of minimum potential energy.

9. Write down the expression of shape function N and displacement u for 1-D bar element.

10. Write down the expression of stiffness matrix for a truss element.

PART B

1. Derive an expression for shape function and assemble the stiffness matrix for bending in beam elements. (16)

2.Derive an expression of shape functions and the stiffness matrix for one dimensional bar element based on global co-ordinate approach. (16)

3.A two noded truss element is shown in the fig. The nodal displacements are u1=5 mm and u2 = 8 mm. Calculate the displacements at x = L/4, L/3 and L/2. (16)

4. A steel plate of uniform thickness 25 mm is subjected to a point load of 420 N at mid depth as shown in fig. The plate is also subjected to self-weight. If E = 2*105 N/mm2 and density = 0.8*10-4 N/mm3. Calculate the displacement at each nodal point and stresses in each element. (16)

5. An axial load of 4*105 N is applied at 30°C to the rod as shown in fig. The temperature is then raised to 60°C. Calculate nodal displacements, stresses in each material and reactions at each nodal point. Take Eal = 0.7*105 N/mm2; Esteel = 2*105 N/mm2 ; αal = 23*10-6 /°C ; αsteel = 12*10-6 /°C. (16)

6.Why higher order elements are needed? Determine the shape functions of an eight noded rectangular element. (16)

7.Derive the stiffness matrix for two dimensional truss elements.(16)

8.Determine the slope and deflection of a cantilever beam subjected to a uniformly distributed load q over the entire span and a point load P acting on its free end.(16)

9.Determine the slope and deflection of a simply supported beam subjected to a uniformly distributed load q over the entire span and a point load P acting at its mid span. (16)

10.Consider the bar as shown in Fig. Calculate the following: (i) Nodal displacements (ii) Element stresses (iii) Support reactions. Take E=200GPa and P=400kN. (16)
11. Consider a taper steel plate of uniform thickness, t=25mm as shown in Fig. The Young's modulus of the plate, E=200GPa and weight density ρ=0.82x10-4 N/mm3. In addition to its self weight, the plate is subjected to a point load P=100N at its mid point. Calculate the following by modeling the plate with two finite elements:

(i) Global force vector {F}. (ii) Global stiffness matrix [K]. (iii) Displacements in each element. (iv) Stresses in each element. (v) Reaction force at the support. (16)

UNIT III

PART A

1. Write down the expression for shape functions for a constant strain triangular element

2. Write a strain displacement matrix for CST element

3. What is LST element?

4. What is QST element?

5. What is meant by plane stress analysis?

6. What is CST element?

7. Write down the stiffness matrix equation for 2-D CST element.

8. Define plane stress analysis.

9. Write a displacement function equation for CST element.

10. Write down the stress-strain relationship matrix for plane stress condition.

PART B

1. A wall of 0.6 m thickness having thermal conductivity of 1.2 W/mK. The wall is to be insulated with a material of thickness 0.06 m having an average thermal conductivity of 0.3 W/mK. The inner surface temperature is 1000 °C and outside of the insulation is exposed to atmospheric air at 30 °C with heat transfer coefficient of 30 W/m2K. Calculate the nodal temperatures (16)

2. A furnace wall is made up of three layers, inside layer with thermal conductivity 8.5 W/mK, the middle layer with conductivity 0.25 W/mK, the outer layer with conductivity 0.08 W/mK. The respective thicknesses of the inner, middle and outer layer are 25 cm, 5 cm and 3 cm resp. The inside temperature of the wall is 600 C and outside of the wall is exposed to atmospheric air at 30 °C with heat transfer coefficient of 45 W/m2K. Determine the nodal temperatures. (16)

3. An aluminium alloy fin of 7 mm thick and 50 mm protrudes from a wall, which is maintained at 120 °C. The ambient air temperature is 22 °C. The heat transfer coefficient and thermal conductivity of the fin material are 140 W/m2K and 55 W/mK respectively. Determine the temperature distribution of fin.(16)

4. A steel rod of diameter d = 2 cm, length L = 5 cm and thermal conductivity k = 50 W/mC is exposed at one end to a constant temperature of 320 °C. The other end is in ambient air of temperature 20 °C with a convection coefficient of h = 100 W/m2°C. Determine the temperature at the midpoint of the rod. (16)

5. Calculate the temperature distribution in a 1-D fin. The fin is rectangular in shape and is 120 mm long, 40 mm wide and 10 mm thick. One end of the fin is fixed and other end is free. Assume that convection heat loss occurs from the end of the fin. Use 2 elements. The temperature at fixed end is 120 °C. Take k = 0.3 W/mm°C; h = 10-3 W/mm2C; T (amb) = 20 °C. (16)

6. A metallic fin, with thermal conductivity k = 360 W/mc, 0.1 cm thick and 10 cm long, extends from a plane wall whose temperature is 235C. Determine the temperature distribution and amount of heat transferred from the fin to the air at 20C with h = 9 W/m2C. Take width of the fin to be 1 m. (16)

7. Evaluate the stiffness matrix for the CST element shown in fig. Assume plane stress condition. Take, t = 20 mm, E = 2*105 N/mm2 and m = 0.25. The coordinates are given in mm.Assemble the strain-displacement matrix for the CST element shown in fig. Take, t = 25 mm and E = 210 Gpa.

8. Derive an expression for shape function for constrain strain triangular element.

9. Determine the shape functions N1, N2 and N3 at the interior point P for the triangular element shown in fig.

UNIT IV

PART A

1. What are the conditions for a problem to be axisymmetric?

2. Give the strain-displacement matrix equation for an axisymmetric triangular element

3. Write down the stress-strain relationship matrix for an axisymmetric triangular element.

4. Write short notes on axisymmetric problems.

5. What are the ways in which a 3-D problem can be reduced to a 2-D approach?

6. What is axisymmetric element?

7. Give the stiffness matrix equation for an axisymmetric triangular element.

8. What are the 4 basic elasticity equations?

9. Write down the displacement equation for an axisymmetric triangular element.

10. Write down the shape functions for an axisymmetric triangular element

PART B

1. The nodal co-ordinates for an axisymmetric triangular are given below: r1 = 15 mm, z1 = 15 mm; r2 = 25 mm, z2 = 15 mm; r3 = 35 mm, z3 = 50 mm. Determine [B] matrix for that element. (16)

2. Evaluate the temperature force vector for the axisymmetric triangular element shown in the fig. The element experiences a 15°C increases in temperature.Take α = 10*10-6/°C, E = 2*105 N/mm2, m = 0.25. (16)

3. Determine the stiffness matrix for the element shown in fig. The co-ordinates are in mm. Take E = 2*105 N/mm2 and m = 0.25. (16)

4. Derive the expression for the element stiffness matrix for an axisymmetric shell element. (16)

5. For the axisymmetric elements shown in fig, determine the element stresses. Let E = 210Gpa and m = 0.25. The co-ordinates are in mm. u1 = 0.05 mm; w1 = 0.03 mm: u2 = 0.02 mm; w2 = 0.02 mm u3 = 0 mm; w3 = 0 mm (16)

6. Derive the strain – displacement matrix [B] for axisymmetric triangular element (16)

7. Derive the stress-strain relationship matrix [D] for the axisymmetric triangular element. (16)

8. Calculate the element stiffness matrix and the thermal force vector for the axisymmetric triangular element shown in fig. The element experiences a 15°C increase in temperature. The coordinates are in mm. Take α = 10*10-6/°C; E = $2*10^5$ N/mm^2; m = 0.25 (16)

9. For the axisymmetric elements shown in fig, determine the element stresses. Let E = 210Gpa and m = 0.25. The co-ordinates are in mm. (16)

The nodal displacements are:u1 = 0.05 mm; w1 = 0.03 mm u2 = 0.02 mm; w2 = 0.02 mm u3 = 0 mm; w3 = 0 mm

UNIT V

PATR A

1. What is an 'Iso-parametric element'?
2. Differentiate between Isoparametric, super parametric and sub parametric
3. Write down the shape functions for 4-noded linear quadrilateral element using natural CO-ORDINATE SYSTEMS
4. What is a 'Jacobian transformation'?
5. What are the advantages of 'Gaussian quadrature' numerical integration for isoparametric elements??
6. How do you calculate the number of Gaussian points in Gaussian quadrature method
7. Find out the number Gaussian points to be considered for _ (x4+3x3-x) dx
8. What is the Jacobian transformation fro a two nodded isoparametric element
9. What is meant by isoparametric formulation?
10. Sketch an general quadrilateral element and an isoparametric quadrilateral element

PART B

1.Derive the element stiffness matrix for a linear isoparametric quadrilateral element (16)

2. Establish the strain – displacement function matrix for the linear quadrilateral element as shown in fig at gauss point r = 0.57735 and s = -0.57735. (16)

3. The integral ∫1-1 (r3 + 2r2 + 1) dr can be evaluated exactly by two point Gaussian quadrature. Examine the effect on the result if three point integration is applied. (16)

4. For a 4 noded rectangular element shown in fig, determine the following:

Jacobian matrix (16)

Strain displacement matrix

Element stresses. Take E = 2*105 N/mm^2 ; m = 0.25;

u = [0,0,0.002,0.003,0.005,0.003,0,0] ; ε = 0 ; η = 0. Assume plane stress condition

5. Derive the shape functions for 4 noded rectangular parent element by using natural coordinate system and coordinate transformation. (16)

6. Evaluate the integral I = ∫1-1 (2 + x + x2) dx using gauss quadrature method and compare with exact solution. (16)

7. Evaluate the integral, I = ∫1-1 cos πx/2 dx by applying 3 point gaussian quadrature and compare with exact solution. (16)

8. Evaluate the integral I = ∫1-1 [3ex + x2 +1/x+2] dx using one point and two point gauss quadrature. Compare with exact solution. (16)

9. For the isoparametric quadrilateral element shown in fig determine the local coordiantes of the point P which has cartesian coordinates (7,4). (16)